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Note

Expeditious Vlasov Solver for Computation of Ion Extraction from a Plasma*

Ion extraction and transport through an accelerating structure have many applications. Recently an algorithm has been developed which is able to solve for the plasma sheath explicitly [1] and has found application for the design of accelerating structures for intense neutral beam injectors for fusion research [2]. Unfortunately, the computational speed of the algorithm makes resource allocation a problem. Described here is an alternative Vlasov solver and Poisson–Vlasov iteration scheme which increases the expeditiousness of the Computation by more than a factor of 10.

The Poisson Vlasov system of equations solved are described in Ref. [3]; in particular, Vlasov's equation for the ions

$$v \cdot \nabla f - \frac{e}{m} \nabla_v f = 0 \tag{1}$$

is solved directly by considering the equations of motion of a specified initial distribution function (see Fig. 1). Previously Hamilton's equations for the ion motion were considered by an ODE solved using a deferred limit integrator [1b, pp. 19–23; 3]. An alternative to this approach is to assume in each cell, for which the Poisson equation is solved, that the electric field is a constant from which it follows that the ion trajectory is a parabola.

This scheme is shown in Fig. 2 for $N = 1$. Since the acceleration interpolator [1b, pp. 29–31; 4], generates accelerations which are linear over a Poisson solution cell, one can obtain useful refinement by subdividing a mesh only for the purpose of orbit refinement. This is shown in Fig. 2 for $N > 1$; utilization of orbit refinement takes place only when, without it, the radial acceleration, a_r, would be greater than $|kv_{z0}^2/z_{\text{cell}}|$, where k is typically 0.01, v_{z0} is the initial axial component of velocity, and z_{cell} = the axial length of a finite difference mesh cell. Using this method of refinement is far less time consuming than increasing the number of mesh points on

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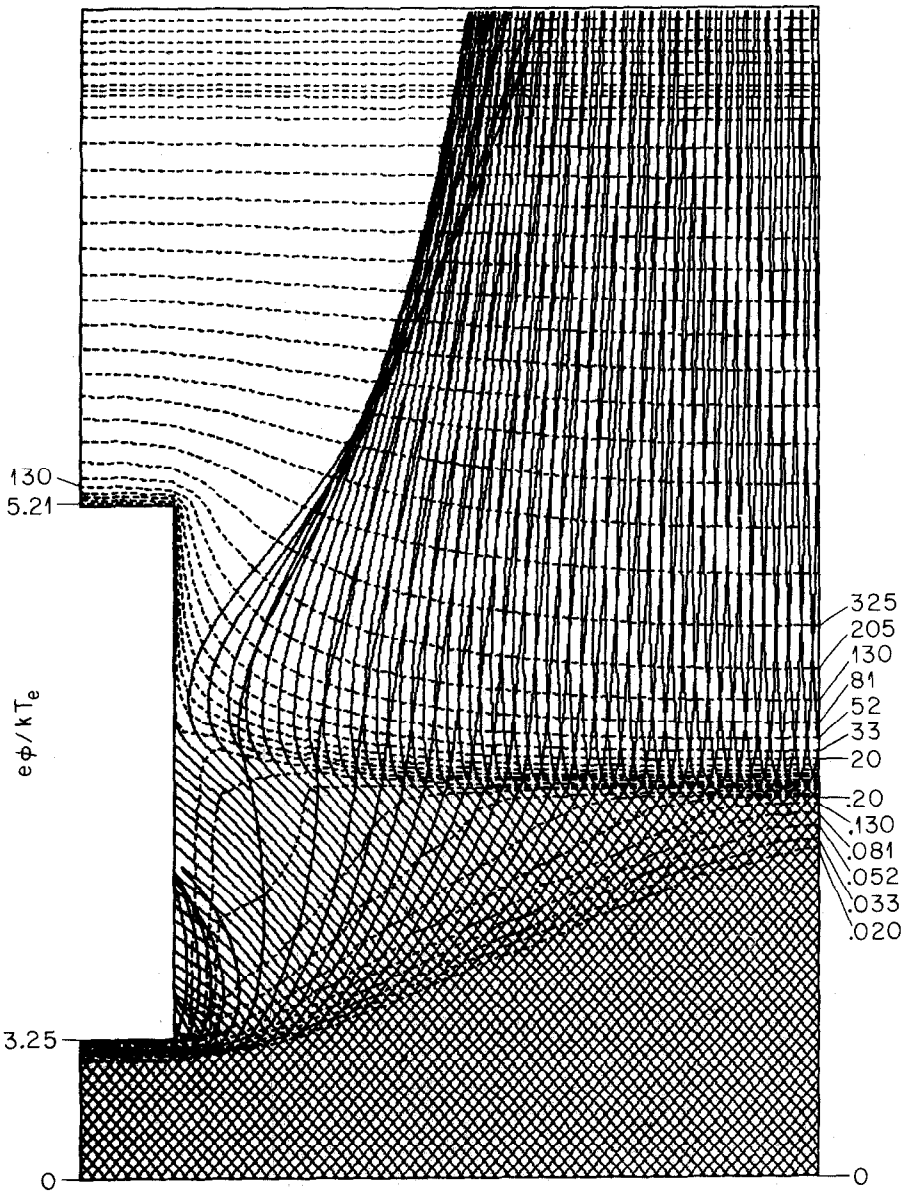

FIG. 1. Typical converged solution for the Poisson–Vlasov system showing ion trajectories by solid lines and equipotential contours by dotted lines. Subspace refinement is only done in those space regions in which a significant deflection occurs.

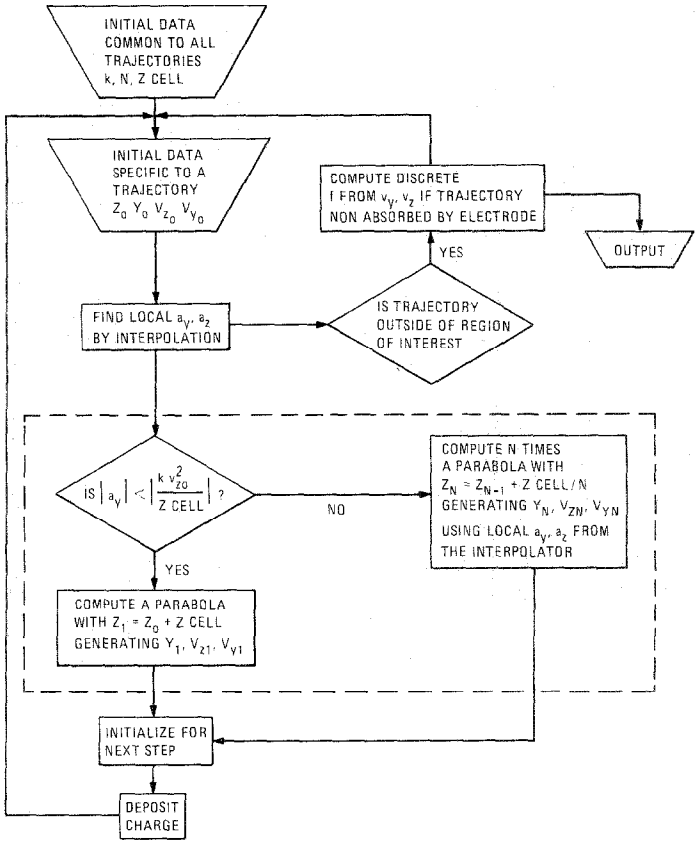

FIG. 2. Flow diagram for new Vlasov solver for the simple case of no orbit reversal.

which the entire Poisson–Vlasov system is considered and is frequently just as effective. For the simplest case where the orbits do not turn around, in the mesh considered, and are traveling in the optic axis direction (z) and within the boundaries, the trajectory is determined from the calculations in the following order:

$$v_{z1} = [(v_{z0})^2 - 2a_z(z_2 - z_1)]^{1/2},$$

$$t = \left| \frac{z_2 - z_1}{v_{z1}} \right| \qquad \text{if} \quad |v_{z1} - v_{z0}| \leqslant v_{z1}/1000,$$

$$t = \left| \frac{v_{z1} - v_{z0}}{a_z} \right| \qquad \text{if} \quad |v_{z1} - v_{z0}| > v_{z1}/1000,$$

$$v_{y1} = v_{y0} + a_y t.$$

Useful values of k and N are 0.01 and 6, respectively, for typical problems we consider. The area enclosed by the dotted lines in Fig. 2 replace the deferred limit integrator in Ref. [1b, pp. 19–23]. The act of replacing the deferred limit integration with parabolas with refinement reduces the trajectory calculation by a factor of over 15 as shown in Fig. 3 by Vlasov computation time. This scheme is not unlike a Vlasov solver used previously [5] without refinement. However, the Poisson solver to which it couples has occasional convergence difficulties, due to the exponentially nonlinear electron contribution, which are claimed to be remedied by the Poisson solver of Refs. [1] or [3].

Because the computation time was dominated by the Vlasov solver with the deferred limit integrator, as shown the case with $n = 40$ in Fig. 3, where 20% of the time is typically spent in the Poisson calculation there was no need to cut corners with the Poisson solver computational time (which has already been reduced substantially in Ref. [1] over that of Refs. [3, 4, 5]). The Poisson–Vlasov iteration scheme for the top two curves in Fig. 3 is shown in Fig. 4 for which convergence of the Poisson equation is obtained before a Vlasov iteration (sequential convergence) [1, 4, 6]. Convergence of the Poisson Vlasov system is considered to obtain when the ion beam divergence, θ, does not change upon further iteration. However, the new Vlasov solver is typically a factor of 15 faster than the old so the same time spent in Poisson computation would amount to about 78% of the computational work. Therefore it is worthwhile examining the Poisson–Vlasov iteration scheme. Using a simultaneous iteration scheme, as shown in Fig. 5, where the Vlasov iteration is made prior to

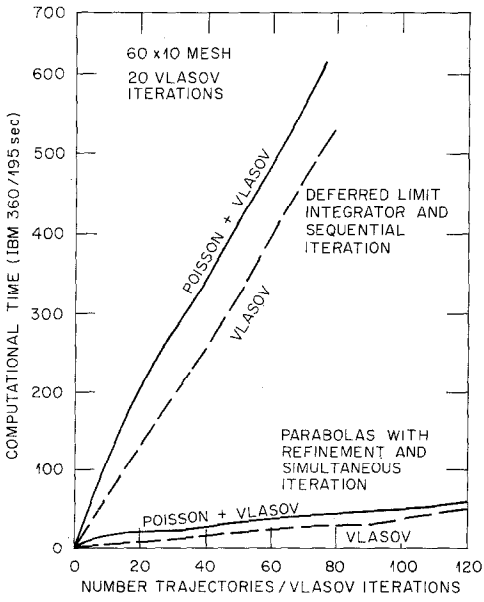

FIG. 3. Computation time, in units of IBM 360/195 seconds, as a function of number of orbits per Vlasov iteration shown for each of two schemes.

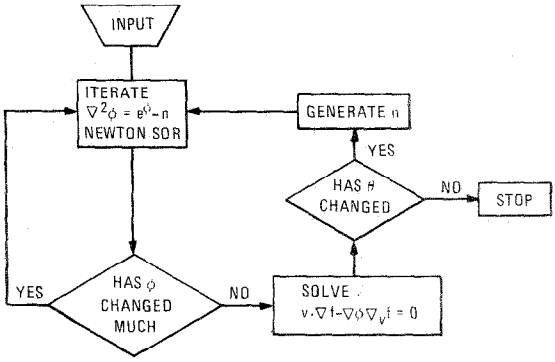

FIG. 4. Flow diagram for sequential Poisson–Vlasov iteration scheme.

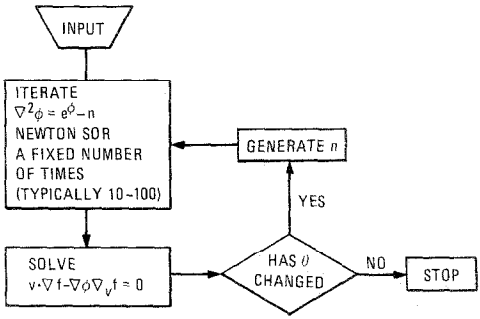

FIG. 5. Flow diagram for simultaneous Poisson–Vlason iteration scheme.

convergence of the Poisson equation, a decrease in computational time of the Poisson solver of a factor of 8 is obtained without any loss in accuracy. The total amount of work the Poisson solver has to do for the iteration scheme shown in Fig. 5 is about that needed for the convergence of the vacuum fields. For the particular case examined above an overall gain of a factor of 12 in increase of computational speed is achieved.

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